User Manual 3.3 Interpolation Methods

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User Manual 3.3 Interpolation Methods

Introduction

Scope

In this section, a focus is realised on the following interpolation methods: spline, bicubic, tricubic, Lagrange and Newton, covariance matrix and linear in 1D, 2D or 3D interpolation.

Javadoc

The interpolation objects are available in the package `org.apache.commons.math3.analysis.interpolation` both in the Commons Math library and the Commons Math Addons library and in the package `org.orekit.propagation.analytical.covariance` in the Orekit library.

Library	Javadoc
Commons Math	Package org.apache.commons.math3.analysis.interpolation
Commons Math Addons	Package org.apache.commons.math3.analysis.interpolation
Orekit	Package org.orekit.propagation.analytical.covariance

Links

None as of now.

Useful Documents

None as of now.

Package Overview

The package `org.apache.commons.math3.analysis.interpolation` contains all the interpolation classes described in this section.

Features Description

Spline interpolation

The ***spline interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the values of x, y. The interpolator gives the function f such as [math]y=f(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double x[] = { 0.0, 1.0, 2.0 };
double y[] = { 1.0, 3.0, 5.0 };

UnivariateInterpolator interpolator = new SplineInterpolator();
UnivariateFunction function = interpolator.interpolate(x, y);
double  value = function .value(0.5);
```

Bicubic interpolation

The ***bicubic interpolator*** generates an interpolating function [math]f(x,y): \mathbb{R}^2 \rightarrow \mathbb{R}[/math]. The interpolator computes internally the coefficients of the bicubic function that is the interpolating function. The user gives as entries 3 sets of values, the values of x, y and z. The interpolator gives the function f such as [math]z=f(x,y)[/math].

For the equation of the plane [math]z=2x-3y + 5[/math]

```
double x[] = { 3, 4, 5, 6.5 };
double y[] = {-4, -3, -1, 2, 2.5 };
double z[][] = {{ 23, 20, 14, 5, 3.5 },
  { 25, 22, 16, 7, 5.5 },
  { 27, 24, 18, 9, 7.5 },
  { 30, 27, 21, 12, 10.5 }};
BivariateGridInterpolator interpolator = new BicubicSplineInterpolator();
BivariateFunction function = interpolator.interpolate(x, y, z);
```

Tricubic interpolation

The ***tricubic interpolator*** generates an interpolating function [math]f(x,y,z): \mathbb{R}^3 \rightarrow \mathbb{R}[/math]. The interpolator computes internally the coefficients of the tricubic function that is the interpolating function. The user gives as entries 4 sets of values, the values of x, y, z and w. The interpolator gives the function f such as [math]w=f(x,y,z)[/math].

For the equation of the plane [math]w=2x- 3y - z + 5[/math]

```
double x[] = { 3.0, 4.0, 5.0, 6.5 };
double y[] = {-4.0, -3.0, -1.0, 2.0, 2.5 };
double z[] = {-12.0, -8.0, -5.5, -3.0, 0.0, 2.5 };
double w[][][] = {{{ 35, 31, 28.5, 26, 23, 20.5 },
{ 32, 28, 25.5, 23, 20, 17.5 },
{ 26, 22, 19.5, 17, 14, 11.5 },
{ 17, 13, 10.5, 8, 5, 2.5 },
{ 15.5, 11.5, 9, 6.5, 3.5, 1 }},
{{ 37, 33, 30.5, 28, 25, 22.5 },
{ 34, 30, 27.5, 25, 22, 19.5 },
{ 28, 24, 21.5, 19, 16, 13.5 },
{ 19, 15, 12.5, 10, 7, 4.5 },
{ 17.5, 13.5, 11, 8.5, 5.5, 3 }},
{{ 39, 35, 32.5, 30, 27, 24.5 },
```

```
{ 36, 32, 39.5, 27, 24, 21.5 },
{ 30, 26, 23.5, 21, 18, 15.5 },
{ 21, 17, 14.5, 12, 9, 6.5 },
{ 19.5, 15.5, 13, 10.5, 7.5, 5 }},
{{ 42, 38, 35.5, 33, 30, 27.5 },
{ 39, 35, 32.5, 30, 27, 24.5 },
{ 33, 29, 26.5, 24, 21, 18.5 },
{ 24, 20, 17.5, 15, 12, 9.5 },
{ 22.5, 18.5, 16, 13.5, 10.5, 8 }}};

TrivariateGridInterpolator interpolator = new TricubicSplineInterpolator();
TrivariateFunction function = interpolator.interpolate(x, y, z, w);
```

Lagrange interpolation

The ***Lagrange interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the values of x, y. The interpolator gives the function f such as [math]y=f(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double x[] = { 0.0, 1.0, 2.0 };
double y[] = { 1.0, 3.0, 5.0 };

UnivariateFunction interpolator = new PolynomialFunctionLagrangeForm(x,y);
double  value = interpolator.value(0.5);
```

Newton interpolation

The ***Newton interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the coefficients [math]c_i[/math]and the centers [math]x_i[/math]such as the polynomial function [math]P(x)=c_0 + c_1 (x - x_0) + ... + c_n (x - x_n)[/math]. The interpolator gives the function f such as [math]y=P(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double c_i[] = { 3.0, 2.0 };
double x_i[] = { 1.0 };

UnivariateFunction interpolator = new PolynomialFunctionNewtonForm(c_i,x_i);
double  value = interpolator.value(0.5);
```

Covariance matrix interpolation

The purpose of this interpolation algorithm is to compute the covariance matrix at a given date through a simplified model of the transition matrix. When a covariance in PV coordinates is searched for an object orbiting around an celestial body, a simple dynamical model can be used, meaning limited to the newtonian attraction, plus a constant acceleration. The value of this constant acceleration will not change the transition matrix.

The transition matrix between a date t_1 and a date t can be approximated :

- at order 0 : by $\phi_1(t_1, t) = I_{3 \times 3}$
- at order 1 : by $\phi_1(t_1, t) = I_{3 \times 3} + J_{PV} (t- t_1)$
- at order 2 : by$\phi_1(t_1, t) =I_{3 \times 3} + J_{PV} (t- t_1)+ 0.5 * J_{PV}^2 (t - t_1)^2$

where $J_{PV} = \left(\begin{array}{cc} 0_{3 \times 3} & I_{3 \times 3} \\ A & 0_{3 \times 3} \end{array} \right)$, $J_{PV}^2 = \left(\begin{array}{cc} A & 0_{3 \times 3} \\ 0_{3 \times 3} & A \end{array} \right)$ and $A =- \frac{ GM}{r^3}\left(I_{3 \times 3} - 3 \frac{ PP^T}{r^2}\right)$, where A is considered as a constant on the interval $[t_1,t]$ and P is the satellite position vector.

We denote by $M(t)$ the covariance matrix at instant t. Let $t \in [t_1,t]$. The transition matrices $\phi_1(t_1, t)$ and $\phi_2(t_2, t)$ are given by the above formula, and since matrix A is constant on $[t_1,t_2]$, we have that the covariance matrix at instant t is given by $M(t) = (1- \alpha) \phi_1(t_1, t) M(t_1)\phi_1^T(t_1, t) + \alpha \phi_2(t_2, t) M(t_2)\phi_2^T(t_2, t),$ with $\alpha = \frac{t-t_1}{t_2-t_1}$.

Linear interpolation

These classes allow linear piecewise interpolations in dimensions 1, 2 or 3.

1D interpolation

Let f be a real function $\mathbb{R} \rightarrow \mathbb{R}$ and $[x_1,x_2]$ the interpolation interval, where $f(x_1),f(x_2)$ are known. For all $x \in [x_1,x_2]$, the interpolated value $f(x)$ is given by $f(x) = f(x_1) + (x-x_1) \frac{f(x_2)- f(x_1)}{x_2-x_1}.$

2D interpolation

The two dimensional interpolation will be two successive 1D interpolations.
Let f be a real function $\mathbb{R}^2 \rightarrow \mathbb{R}$ and $[x_1,x_2] \times [y_1,y_2]$ the interpolation interval.
First, a 1D interpolation in the y direction is made, leading to $f(x,y_1) = f(x_1,y_1) + (y-y_1) \frac{f(x_2,y_1)- f(x_1,y_1)}{y_2-y_1},$
$f(x,y_2) = f(x_1,y_2) + (y-y_1) \frac{f(x_2,y_2)- f(x_1,y_2)}{y_2-y_1}.$

Then a second 1D interpolation is made in the x direction with the previous two interpolated values : $f(x,y) = f(x,y_1) + (x-x_1) \frac{f(x,y_2)- f(x, y_1)}{x_2-x_1}.$

3D interpolation

Let f be a real function $\mathbb{R}^3 \rightarrow \mathbb{R}$ and $[x_1,x_2] \times [y_1,y_2] \times [z_1,z_2]$ the interpolation interval. There will be

[math]2^3- 1[/math] successives 1D interpolations.

[math]f(x,y,z)[/math] is interpolated from [math]f(x,y,z_1)[/math] and [math]f(x,y,z_2)[/math].

[math]f(x,y,z_1)[/math] is interpolated from [math]f(x,y_1,z_1)[/math] and [math]f(x,y_2,z_1)[/math].
[math]f(x,y,z_2)[/math] is interpolated from [math]f(x,y_1,z_2)[/math] and [math]f(x,y_2,z_2)[/math].

[math]f(x,y_1,z_1)[/math] is interpolated from [math]f(x_1,y_1,z_1)[/math] and [math]f(x_2,y_1,z_1)[/math].
[math]f(x,y_2,z_1)[/math] is interpolated from [math]f(x_1,y_2,z_1)[/math] and [math]f(x_2,y_2,z_1)[/math].
[math]f(x,y_1,z_2)[/math] is interpolated from [math]f(x_1,y_1,z_2)[/math] and [math]f(x_2,y_1,z_1)[/math].
[math]f(x,y_2,z_2)[/math] is interpolated from [math]f(x_1,y_2,z_2)[/math] and [math]f(x_2,y_2,z_2)[/math].

Getting Started

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Interfaces

The library defines the following interfaces related to interpolation :

Interface	Summary	Javadoc
UnivariateInterpolator	Interface for a univariate interpolating function.	...
BivariateGridInterpolator	Interface for a bivariate interpolating function where the sample points must be specified on a regular grid.	...
TrivariateGridInterpolator	Interface for a trivariate interpolating function where the sample points must be specified on a regular grid.	...
UnivariateFunction	Interface for a univariate function	...

Classes

This section is about the following classes related to interpolation :

In Commons Math

Class	Summary	Javadoc
SplineInterpolator	Spline interpolator for a univariate real function.	...
BicubicSplineInterpolator	Bicubic spline interpolator for a bivariate real function.	...
TricubicSplineInterpolator	Tricubic spline interpolator for a trivariate real function.	...

PolynomialFunctionLagrangeForm	Lagrange interpolator, directly usable as a univariate real function.	...
PolynomialFunctionNewtonForm	Newton interpolator, directly usable as a univariate real function.	...

In Orekit

Class	Summary	Javadoc
CovarianceInterpolation	Interpolator of a covariance matrix based on its two surrounding covariance matrices.	...
OrbitCovariance	Class containing a covariance matrix and its associated AbsoluteDate. New class replacing older class CovarianceMatrix	...

In Commons Math Addons

Class	Summary	Javadoc
AbstractLinearIntervalsFunction	Abstract class for linear interpolations.	...
UniLinearIntervalsFunction	Linear one-dimensional function.	...
BiLinearIntervalsFunction	Linear two-dimensional function.	...
TriLinearIntervalsFunction	Linear three-dimensional function.	...
UniLinearIntervalsInterpolator	Interpolator of linear one-dimensional functions.	...
BiLinearIntervalsInterpolator	Interpolator of linear two-dimensional functions.	...
TriLinearIntervalsInterpolator	Interpolator of linear three-dimensional functions.	...

Tutorials

Tutorial 1

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Tutorial 2

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Tips & Tricks

None as of now.

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- Main differences between V4.9 and V4.8
- Main differences between V4.8 and V4.7
- Main differences between V4.7 and V4.6.1

- Main differences between V4.6.1 and V4.5.1
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