User Manual 4.2 Interpolation Methods

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User Manual 4.2 Interpolation Methods

Introduction

Scope

In this section, a focus is realised on the following interpolation methods: spline, bicubic, tricubic, Lagrange and Newton, covariance matrix and linear in 1D, 2D or 3D interpolation.

Javadoc

The interpolation objects are available in the package `fr.cnes.sirius.patrius.math.analysis.interpolation` and in the package `fr.cnes.sirius.patrius.propagation.analytical.covariance`.

Library	Javadoc
Patrius	Package fr.cnes.sirius.patrius.math.analysis.interpolation
Patrius	Package fr.cnes.sirius.patrius.math.analysis.interpolation
Patrius	Package fr.cnes.sirius.patrius.propagation.analytical.covariance

Links

None as of now.

Useful Documents

None as of now.

Package Overview

The package `fr.cnes.sirius.patrius.math.analysis.interpolation` contains all the interpolation classes described in this section.

Features Description

Spline interpolation

The ***spline interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the values of x, y. The interpolator gives the function f such as [math]y=f(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double x[] = { 0.0, 1.0, 2.0 };
```

```
double y[] = { 1.0, 3.0, 5.0 };

UnivariateInterpolator interpolator = new SplineInterpolator();
UnivariateFunction function = interpolator.interpolate(x, y);
double  value = function .value(0.5);
```

Bicubic interpolation

The ***bicubic interpolator*** generates an interpolating function [math]f(x,y): \mathbb{R}^2 \rightarrow \mathbb{R}[/math]. The interpolator computes internally the coefficients of the bicubic function that is the interpolating function. The user gives as entries 3 sets of values, the values of x, y and z. The interpolator gives the function f such as [math]z=f(x,y)[/math].

For the equation of the plane [math]z=2x-3y + 5[/math]

```
double x[] = { 3, 4, 5, 6.5 };
double y[] = {-4, -3, -1, 2, 2.5 };
double z[][] = {{ 23, 20, 14, 5, 3.5 },
  { 25, 22, 16, 7, 5.5 },
  { 27, 24, 18, 9, 7.5 },
  { 30, 27, 21, 12, 10.5 }};
BivariateGridInterpolator interpolator = new BicubicSplineInterpolator();
BivariateFunction function = interpolator.interpolate(x, y, z);
```

Tricubic interpolation

The ***tricubic interpolator*** generates an interpolating function [math]f(x,y,z): \mathbb{R}^3 \rightarrow \mathbb{R}[/math]. The interpolator computes internally the coefficients of the tricubic function that is the interpolating function. The user gives as entries 4 sets of values, the values of x, y, z and w. The interpolator gives the function f such as [math]w=f(x,y,z)[/math].

For the equation of the plane [math]w=2x- 3y - z + 5[/math]

```
double x[] = { 3.0, 4.0, 5.0, 6.5 };
double y[] = {-4.0, -3.0, -1.0, 2.0, 2.5 };
double z[] = {-12.0, -8.0, -5.5, -3.0, 0.0, 2.5 };
double w[][][] = {{{ 35, 31, 28.5, 26, 23, 20.5 },
{ 32, 28, 25.5, 23, 20, 17.5 },
{ 26, 22, 19.5, 17, 14, 11.5 },
{ 17, 13, 10.5, 8, 5, 2.5 },
{ 15.5, 11.5, 9, 6.5, 3.5, 1 }},
{{ 37, 33, 30.5, 28, 25, 22.5 },
{ 34, 30, 27.5, 25, 22, 19.5 },
{ 28, 24, 21.5, 19, 16, 13.5 },
{ 19, 15, 12.5, 10, 7, 4.5 },
{ 17.5, 13.5, 11, 8.5, 5.5, 3 }},
{{ 39, 35, 32.5, 30, 27, 24.5 },
{ 36, 32, 39.5, 27, 24, 21.5 },
```

```
{ 30, 26, 23.5, 21, 18, 15.5 },
{ 21, 17, 14.5, 12, 9, 6.5 },
{ 19.5, 15.5, 13, 10.5, 7.5, 5 }},
{{ 42, 38, 35.5, 33, 30, 27.5 },
{ 39, 35, 32.5, 30, 27, 24.5 },
{ 33, 29, 26.5, 24, 21, 18.5 },
{ 24, 20, 17.5, 15, 12, 9.5 },
{ 22.5, 18.5, 16, 13.5, 10.5, 8 }}};

TrivariateGridInterpolator interpolator = new TricubicSplineInterpolator();
TrivariateFunction function = interpolator.interpolate(x, y, z, w);
```

Lagrange interpolation

The ***Lagrange interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the values of x, y. The interpolator gives the function f such as [math]y=f(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double x[] = { 0.0, 1.0, 2.0 };
double y[] = { 1.0, 3.0, 5.0 };

UnivariateFunction interpolator = new PolynomialFunctionLagrangeForm(x,y);
double  value = interpolator.value(0.5);
```

Newton interpolation

The ***Newton interpolator*** generates an interpolating function [math]f(x): \mathbb{R} \rightarrow \mathbb{R}[/math]. The user gives as entries 2 sets of values, the coefficients [math]c_i[/math]and the centers [math]x_i[/math]such as the polynomial function [math]P(x)=c_0 + c_1 (x - x_0) + ... + c_n (x - x_n)[/math]. The interpolator gives the function f such as [math]y=P(x)[/math].

For the linear equation [math]y=2x+1[/math]

```
double c_i[] = { 3.0, 2.0 };
double x_i[] = { 1.0 };

UnivariateFunction interpolator = new PolynomialFunctionNewtonForm(c_i,x_i);
double  value = interpolator.value(0.5);
```

Covariance matrix interpolation

The purpose of this interpolation algorithm is to compute the covariance matrix at a given date through a simplified model of the transition matrix. When a covariance in PV coordinates is searched for an object orbiting around an celestial body, a simple dynamical model can be used, meaning limited to the newtonian attraction, plus a constant acceleration. The value of this constant acceleration will not change the transition matrix.

The transition matrix between a date t_1 and a date t can be approximated :

- at order 0 : by $\phi_1(t_1, t) = I_{3 \times 3}$
- at order 1 : by $\phi_1(t_1, t) = I_{3 \times 3} + J_{PV} (t- t_1)$
- at order 2 : by$\phi_1(t_1, t) =I_{3 \times 3} + J_{PV} (t- t_1)+ 0.5 * J_{PV}^2 (t - t_1)^2$

where $J_{PV} = \left(\begin{array}{cc} 0_{3 \times 3} & I_{3 \times 3} \\ A & 0_{3 \times 3} \end{array} \right)$, $J_{PV}^2 = \left(\begin{array}{cc} A & 0_{3 \times 3} \\ 0_{3 \times 3} & A \end{array} \right)$ and $A =- \frac{ GM}{r^3}\left(I_{3 \times 3} - 3 \frac{ PP^T}{r^2}\right)$, where A is considered as a constant on the interval $[t_1,t]$ and P is the satellite position vector.

We denote by $M(t)$ the covariance matrix at instant t. Let $t \in [t_1,t]$. The transition matrices $\phi_1(t_1, t)$ and $\phi_2(t_2, t)$ are given by the above formula, and since matrix A is constant on $[t_1,t_2]$, we have that the covariance matrix at instant t is given by $M(t) = (1- \alpha) \phi_1(t_1, t) M(t_1)\phi_1^T(t_1, t) + \alpha \phi_2(t_2, t) M(t_2)\phi_2^T(t_2, t),$ with $\alpha = \frac{t-t_1}{t_2-t_1}$.

Linear interpolation

These classes allow linear piecewise interpolations in dimensions 1, 2 or 3.

1D interpolation

Let f be a real function $\mathbb{R} \rightarrow \mathbb{R}$ and $[x_1,x_2]$ the interpolation interval, where $f(x_1),f(x_2)$ are known. For all $x \in [x_1,x_2]$, the interpolated value $f(x)$ is given by $f(x) = f(x_1) + (x-x_1) \frac{f(x_2)- f(x_1)}{x_2-x_1}.$

2D interpolation

The two dimensional interpolation will be two successive 1D interpolations. Let f be a real function $\mathbb{R}^2 \rightarrow \mathbb{R}$ and $[x_1,x_2] \times [y_1,y_2]$ the interpolation interval. First, a 1D interpolation in the y direction is made, leading to $f(x,y_1) = f(x_1,y_1) + (y-y_1) \frac{f(x_2,y_1)- f(x_1,y_1)}{y_2-y_1},$

$f(x,y_2) = f(x_1,y_2) + (y-y_1) \frac{f(x_2,y_2)- f(x_1,y_2)}{y_2-y_1}.$

Then a second 1D interpolation is made in the x direction with the previous two interpolated values : $f(x,y) = f(x,y_1) + (x-x_1) \frac{f(x,y_2)- f(x, y_1)}{x_2-x_1}.$

3D interpolation

Let f be a real function $\mathbb{R}^3 \rightarrow \mathbb{R}$ and $[x_1,x_2] \times [y_1,y_2] \times [z_1,z_2]$ the interpolation interval. There will be

[math]2^3- 1[/math] successives 1D interpolations.

[math]f(x,y,z)[/math] is interpolated from [math]f(x,y,z_1)[/math] and [math]f(x,y,z_2)[/math].

[math]f(x,y,z_1)[/math] is interpolated from [math]f(x,y_1,z_1)[/math] and [math]f(x,y_2,z_1)[/math].

[math]f(x,y,z_2)[/math] is interpolated from [math]f(x,y_1,z_2)[/math] and [math]f(x,y_2,z_2)[/math].

[math]f(x,y_1,z_1)[/math] is interpolated from [math]f(x_1,y_1,z_1)[/math] and [math]f(x_2,y_1,z_1)[/math].

[math]f(x,y_2,z_1)[/math] is interpolated from [math]f(x_1,y_2,z_1)[/math] and [math]f(x_2,y_2,z_1)[/math].

[math]f(x,y_1,z_2)[/math] is interpolated from [math]f(x_1,y_1,z_2)[/math] and [math]f(x_2,y_1,z_1)[/math].

[math]f(x,y_2,z_2)[/math] is interpolated from [math]f(x_1,y_2,z_2)[/math] and [math]f(x_2,y_2,z_2)[/math].

Getting Started

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Interfaces

The library defines the following interfaces related to interpolation :

Interface	Summary	Javadoc
UnivariateInterpolator	Interface for a univariate interpolating function.	...
BivariateGridInterpolator	Interface for a bivariate interpolating function where the sample points must be specified on a regular grid.	...
TrivariateGridInterpolator	Interface for a trivariate interpolating function where the sample points must be specified on a regular grid.	...
UnivariateFunction	Interface for a univariate function	...

Classes

This section is about the following classes related to interpolation :

Class	Summary	Javadoc
SplineInterpolator	Spline interpolator for a univariate real function.	...
BicubicSplineInterpolator	Bicubic spline interpolator for a bivariate real function.	...
TricubicSplineInterpolator	Tricubic spline interpolator for a trivariate real function.	...

PolynomialFunctionLagrangeForm	Lagrange interpolator, directly usable as a univariate real function.	...
PolynomialFunctionNewtonForm	Newton interpolator, directly usable as a univariate real function.	...

Class	Summary	Javadoc
CovarianceInterpolation	Interpolator of a covariance matrix based on its two surrounding covariance matrices.	...
OrbitCovariance	Class containing a covariance matrix and its associated AbsoluteDate. New class replacing older class CovarianceMatrix	...

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Class	Summary	Javadoc
AbstractLinearIntervalsFunction	Abstract class for linear interpolations.	...
UniLinearIntervalsFunction	Linear one-dimensional function.	...
BiLinearIntervalsFunction	Linear two-dimensional function.	...
TriLinearIntervalsFunction	Linear three-dimensional function.	...
UniLinearIntervalsInterpolator	Interpolator of linear one-dimensional functions.	...
BiLinearIntervalsInterpolator	Interpolator of linear two-dimensional functions.	...
TriLinearIntervalsInterpolator	Interpolator of linear three-dimensional functions.	...

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- Dernière modification de cette page le 3 juin 2019 à 14:43.